Frontrunning the signals: As arbitrage between sophisticates

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This paper presents a model in which some sophisticated investors do not wait for receipt of a signal before purchasing an asset. Its critical innovation is an arbitrage equation for frontrunning. Some sophisticates who will receive information in the next period arbitrage against similar sophisticates who will act on that information in that next period when the information is received. The costs of such frontrunning are borne totally by unsophisticated traders—with no gain or loss to sophisticates. Nor does the frontrunning produce any information discovery. Thus, this paper describes a financial-market anomaly: of inefficient financial transactions with gains to no one.

frontrunning | rent seeking | financial markets

This paper develops a model, whose key feature is a special form of frontrunning, in which sophisticated traders purchase assets in advance of a signal that will be uniformly received by all sophisticates. The price of the asset before receipt of the signal then exactly equalizes the returns to two strategies. One strategy is to buy now; keep the asset if the signal is positive/dump onto unsophisticated traders if the signal is negative. The other strategy is to wait for the signal; then buy the asset if the signal is positive/do not buy if negative. In the model the added transaction costs of buying the asset and selling it if the signal is negative are all absorbed by the unsophisticated traders. Yet, even though the unsophisticated traders are paying for all of the costs of the frontrunning, there are still no gains at all from this trading activity to the sophisticates. (In this paper we use "frontrunning" according to a general vernacular usage: as preemptive action that anticipates similar action by others; this is in the same spirit as—but much more general than—the more specific use of the term in finance as "trading before other traders, based on specific information about the direction in which other traders will trade in the future").*

While the model in this paper is very special, it raises a question of considerable generality. There is a significant literature regarding the surprising rise in the Gross-Domestic-Product share of the financial sector [Philippon (1)]. Zingales (2) has been similarly concerned with excessive rent seeking in financial markets. Turner (ref. 3, p. 44) has said, further, that "financial activity [goes] beyond those [that] deliver true social value.... Numerous studies have shown that much active asset management adds no value but does add significant cost." He has also described the huge increase in "intrafinancial" transactions. In casting light on a special case of such transactions, with no gains to those who instigate increased trading, this paper then poses the question, whether there are not many examples of this ilk: in which the returns to the arbitrageurs themselves are negligible, but the costs of this much ado about nothing are borne by unsophisticated others on the sidelines.

This paper is based on a three-period barebones model with both sophisticates and unsophisticated traders and also with short-sale constraints. In period 2 the value of the asset is revealed to sophisticated traders; in period 3 its value becomes known to everyone. With transaction costs below a threshold, in period 1 some sophisticated traders will purchase the asset in anticipation of the signal that they will receive in the following period. If the signal in period 2 is positive, they will keep the asset; if it is negative, they will dump it onto the unsophisticated traders. The returns to sophisticates are exactly the same as in the corresponding model, in which trades can only occur in periods 2 and 3, after the revelation of the value of the asset to the sophisticates.

The model in this paper thus describes a type of frontrunning absent from previous papers. The literature on frontrunning of which we are aware—notably including Lewis' *Flash Boys* (4)—concerns the information advantage of the frontrunners. In those papers frontrunners obtain information in advance of their competitors. That is not what is happening here: These sophisticated frontrunners in period 1 do not have advance information relative to their fellow sophisticates, who will be their competitors in the next period if the signal is positive. Instead, they are making those early purchases (in period 1) to obtain a better price for the asset in anticipation that their sophisticated competitors will bid up that price if the signal is positive. In this case the price in period 1 will just settle at the margin at which the returns to two strategies are exactly equalized; that is, the returns to buy now/dump later in the event of a negative signal are exactly equal to the returns to wait for the signal/buy if positive.

The returns to the sophisticates are totally independent of the existence, or nonexistence, of this equilibrium frontrunning. Why so? Because the returns to the sophisticated buyers are

Significance

This paper adds another dimension to the analysis of equilibria in markets with sophisticated traders. There is a rat race between sophisticated investors, who purchase assets in advance of receiving information. Thereby they avoid the loss due to waiting too long if the information turns out positive, as they also take advantage of the opportunity to dump if the information is negative. The rat race results in a special result: All of the transaction costs of the extra frontrunning are borne by the unsophisticated traders, with no gain to the sophisticates. This paper hence provides a specific instance of inefficient financial transactions and excessive rent seeking with gains to no one.

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*We are grateful to one of our referees for suggesting this common usage in finance; he also pointed out that "in [this] paper, the front-running is always buying, and the direction they trade in period one is by construction not correlated with anything that happens in period two."

anchored: since the number of sophisticates in period 2 who hold the asset will be the same, irrespective of the purchases in period 1. Specifically, the exact same number of sophisticates will hold the asset if the signal is positive; no sophisticates will hold it if the signal is negative (since the period 1 buyers will dump it, and no one else will buy it). Hence the price of the asset in period 2 is anchored. But that means, in turn, that the unsophisticated traders will pay all of the transaction costs involved in the arbitrage.[†]

Our barebones model also yields some additional results: 1) prices in excess of fundamentals, which are greater with higher probability of a positive signal; 2) increases in losses with increase in the number of sophisticates; and 3) amplification of frontrunning with sequential signals.

This paper proceeds as follows. *Review of Literature* reviews the literature. *The Model* presents a model of frontrunning. *Transaction Costs* analyzes the effects of transaction costs and calculates the losses to unsophisticated traders caused by frontrunning. *Sequential Signals* examines the effects of sequential signals. *Conclusion* concludes.

Review of Literature

There has been a huge literature on models with sophisticates and unsophisticated traders since the pioneering articles by Kyle (5) and DeLong et al. (6, 7). However, we are unaware of any model in this literature which explicitly addresses whether a future signal results in competition among the sophisticates to frontrun each other. This paper therefore suggests in simple stark fashion a penumbra of considerations regarding the effects of competition by sophisticates, not just with unsophisticated traders, but with their fellow sophisticates to frontrun each other, knowing that future signals will occur, with their trading governed by an arbitrage condition among sophisticated traders.

The closest precedent to our paper, in spirit and modeling, is Shleifer and Vishny (8). Like us, they also have an equation where an arbitrageur will decide whether to trade in an initial period or in the subsequent one. But their mechanism is different from ours. For them, the trading in the initial period was to benefit from the mispricing in that period from fundamentals; absent such initial-period mispricing, no arbitrage trading will occur. In contrast, absent initial-period frontrunning, in our model there would be no deviation of prices from fundamentals in that initial period. Instead, the frontrunners have a different motivation. They are reacting to the fact that sophisticated investors in the next period will have superior information about fundamentals; and they are bidding according to their expectations regarding what prices will be then.

Such differences notwithstanding, this paper owes a great deal to Shleifer and Vishny (8), since it is a further example of a very important principle that, at the minimum, was much clarified there. The major action in unsophisticated trader models is not just from the competition between sophisticates and unsophisticated traders (with the sophisticates typically winning, and the unsophisticated traders typically losing). Instead, major action can come from the competition between the sophisticates themselves. A contribution of our frontrunning model is to illustrate this principle in a different context: with its parsimonious representation of the effects of the role of competition among sophisticates in the determination of pricing in a model of frontrunning.

In our model as the number of sophisticated traders rises, there are unambiguous losses. Thus, this paper also relates to the effect of rational speculators on asset prices. According to common wisdom, such speculators are stabilizing: moving prices toward fundamentals (5, 9–13). But, in dissent, economists have also shown how large speculators can profitably drive prices away from fundamentals with market manipulations (for example, refs. 14–16). That still leaves the question of the role of small-scale speculators in competitive markets. In this regard, DeLong et al. (6) have shown how competitive speculators can push price away from fundamentals—profiting from sales to feedback traders who overreact to price movements. In contrast, our model also has competitive speculators, but with a different mechanism: Asset prices are pushed above fundamentals because of the frontrunning by sophisticated traders.

Our paper also contributes to the literature pioneered by Harrison and Kreps (17) and Scheinkman and Xiong (18); they showed that, in difference-of-opinion models with short-sale constraints, overconfidence of traders can result in asset prices in excess of fundamentals.[‡] In these models, trade occurs because of the difference of opinion between optimists and pessimists regarding asset values. As optimists and pessimists sequentially switch their positions over time, a bubble can arise with short-sale constraints. In those models, trade occurs due to the tension between optimists and pessimists. Instead, in our model the frontrunning occurs because of the competition among sophisticated traders themselves, all of whom have the same information and valuation of the asset. Furthermore, while the difference-of-opinion models with short-sale constraints have been applied to explain excess price and trade volume before news announcements (such as on earnings), our model of frontrunning purchases provides additional reason for the "intriguing pattern" (ref. 20, p. 26) observed by Lamont and Frazzini that institutional traders tend to initiate abnormally large net purchases prior to announcements of earnings.[§] Our paper therefore complements and expands the insights of difference-of-opinion models.

This paper also gives a different perspective on complementarity/substitutability in informed trading and information acquisition (25–30). For example, Goldstein and Yang (30) analyze strategic complementarities in trading and information acquisition. They illustrate that trading on one signal may reduce uncertainty in trading on another signal and hence encourages more trading and information acquisition on the second signal. Our paper has a different channel. In our model, all traders are risk neutral, so the uncertainty channel is absent. But the existence of the second signal increases the trading volume for the first signal, as sophisticates with access to the first signal will not only frontrun the first signal; they also frontrun the frontrunning of the sophisticates who will later have access to the second signal.[¶]

The Model

We present a simple three-period model in which sophisticated traders have an incentive to frontrun each other. We first

[†]Our model focuses on the pecuniary gains and losses to sophisticated and unsophisticated traders. We have not specifically modeled their utility functions from trading. If one introduces utility gains and losses, then there can be various welfare interpretations based on different unmodeled assumptions related to how trading benefits or harms traders.

[‡]See also Scheinkman (19) for detailed review.

[§]Another explanation for this pattern is the attention-grabbing theory as in refs. 20–22. Relatedly, some theories have also been proposed for explaining pre-Federal Open Market Committee excess equity returns, such as the required risk premium for bearing systematic risk or the reallocation of risk due to time-varying market participation (23, 24).

[¶]Pagnotta and Philippon (31) also analyze how traders choose among trading venues based on the provision of trading services (i.e., speed) and fees. They find that speed-sensitive investors tend to trade in faster venues, which, in equilibrium, charge higher fees. Instead, we examine whether, and how, sophisticates trade prior to the receipt of a private signal.

describe the assumptions on the timing, payoff, and demand functions and then describe the frontrunning equilibrium.

Assumptions.

Assets, periods, and payoffs. There are two assets and three periods. The payoff, θ, to the first of these assets is drawn from a two-point distribution: $(1+h)$ with probability f and $(1-l)$ with probability $(1-f)$.

In period 1, sophisticated traders know that they will obtain information in period 2 on the true value of θ. Then in period 3, this true value of θ will be publicly revealed.

The total supply of the risky asset is fixed at 1. The second asset is cash.

Traders and trading. There are two types of traders: unsophisticated traders and sophisticated traders. In period 2, sophisticated traders receive the private signal regarding whether the asset quality is high (h) or low (l).

At the beginning of period 1, all assets are held by unsophisticated traders.[#] In this period, sophisticated traders know the distribution of θ, which will be revealed to them (but not to unsophisticated traders) in period 2.

Each sophisticated trader has the resources to buy at most one share. Thus, the trader can purchase the asset in period 2 if the trader has not made a previous purchase in period 1. There is also a short-sale constraint so that the trader cannot sell the asset if the trader does not own it. But if the trader has purchased the asset in period 1, the trader can sell it in period 2.

All traders are risk neutral. Hence traders purchase the asset based on expected returns. If the expected returns to an asset share are positive, they will buy a share; if they are negative, they will not.

Demand of unsophisticated traders. We describe the demand for the asset by types of trader, in turn.

The total demand D^u by unsophisticated traders in periods 1 and 2 is

$$D^u = 1 - b(p - \hat{\theta}), \qquad [1]$$

where p is the market price.

The value of $\hat{\theta}$ in periods 1 and 2 in [**1**] is

$$\hat{\theta} = f(1+h) + (1-f)(1-l); \qquad [2]$$

i.e., the right-hand side of [**2**] is the expected value of the realization of θ in period 3. As one form of unsophisticated traders' naïveté, they do not update their expectation of θ, in either period 1 or period 2 (for example, dependent on the price or trade volume).[‖]

Such a demand curve can be derived, for example, from assuming that the unsophisticated traders have a distribution of beliefs regarding the value of the asset; those beliefs do not change between periods 1 and 2. In this paper, we do not provide a specific microfoundation for this demand curve: We simply take it as a reduced form (which could come from many possible detailed microspecifications). This parsimony allows our paper to focus on its central point: the resultant arbitrage between the trading by sophisticates prior to receipt of a private signal and their trading after receipt of the signal.

There is empirical evidence for the existence of such downward-sloping demand curves in financial markets, just three examples being refs. 33 and 34 on stock markets and ref. 35 on currency markets. Proposed explanations include, for example, illiquidity (36–38), asymmetric information (5, 11, 12, 39), and imperfect substitution across stocks (40).

An additional assumption further contributes to expositional simplicity. In period 2, we assume that unsophisticated traders do not update their demand curve.[**] This could be due to the naïveté of the unsophisticated traders, such as due to inattention to the behavior of sophisticates (23, 41, 42).[††] It has also been suggested to us that another possible source for such a stable demand curve could come from rational traders: They might be central banks that are leaning against the wind to provide market liquidity to smooth financial markets; they are willing to take the resultant losses to accomplish their stabilization goals.[‡‡]

This simple demand function **1** of unsophisticated traders means that, in period 1, in the absence of sophisticated traders, unsophisticated traders will demand the asset so that its price will be equal to its expected fundamental value.

Demand of sophisticated traders. Sophisticated traders can decide to hold one unit of the asset in all three periods; they can sell what they own in periods 2 and 3. They can be partitioned into three groups according to their trading strategies:

1) Group 1. A member of this group purchases the asset in period 1. In period 2, after receiving the private signal, the trader will keep the asset if the price in period 3 will be greater than or equal to the price in period 2; and the trader will sell otherwise.
2) Group 2. A member of this group does not purchase the asset in period 1. In period 2, the trader will buy the asset if the price is less than or equal to its fundamental value in the next period. But, because of the short-sale constraint, the trader cannot sell it.
3) Group 3. A member of this group purchases the asset neither in period 1 nor in period 2.

Fig. 1 summarizes the timeline of signals and trades by group.

We further make the following assumptions that 1) the total number of sophisticated traders over all groups is N; 2) markets are competitive (e.g., no traders have market power); 3) for simplicity of solution, the market rate of interest is zero; 4) total demand is the sum of the demand by sophisticated traders and by unsophisticated traders; 5) in equilibrium the market clears so that supply equals demand; and 6) in period 3, in which all uncertainty is resolved, and in which unsophisticated traders and sophisticates have the exact same information with certainty, the price is equal to the fundamental value associated with the publicly announced signal.

[#] This corresponds to the assumption in (32) that sophisticated traders hold no assets at the beginning of the trading period. It has little effect on the qualitative results of our model.

[‖] With small h and l, then the expected value for the asset perceived at period 1, $\hat{\theta}(=f(1+h)+(1-f)(1-l))$, will be close to 1. In this case, the demand curve for unsophisticated traders, $D^u = 1 - b(p - \hat{\theta})$, could also be regarded as a local log-linear approximation of a constant-elasticity demand function of $D = P^{-b}$. Then a 1% increase of price P could be interpreted as being associated with a b% decrease of demand D. In this way, the impact of parameters on D and P can be interpreted as percentage changes and will hence be approximately dimensionless.

[**] Even if the demand function is updated in period 2, to the extent that the existence of unsophisticated traders allows a sophisticate to make positive profits from frontrunning, our mechanism will still be relevant (see *Transaction Costs* and *SI Appendix*, section E for more details).

[††] When the price rises at period 2, the responses of the unsophisticated traders can also be consistent with a disposition effect, where they sell shares quickly to take small profits without achieving the full potential profits (43–45). On the other hand, in some cases, despite the decline of asset value, unsophisticated traders may still buy the asset aggressively. For example, during the Chinese stock market crash around June 2015, unsophisticated retail investors were spurred by the government to buy stocks as prices fell, which they followed, even with margin purchases, to prop up the prices (ref. 46 and https://www.npr.org/2015/08/27/435113627/china-s-government-encouraged-ordinary-investors-to-make-risky-margin-bets).

[‡‡] Similarly, the frontrunning model might also be applicable to commodity price stabilization schemes. The literature on the schemes has examined their impacts on demand, storage, supply, and price volatility (47–50); but it has not explored whether and how speculators may frontrun the private signals on commodities to profit from the stabilization schemes. We are especially grateful to one of our referees for pointing out these two applications.

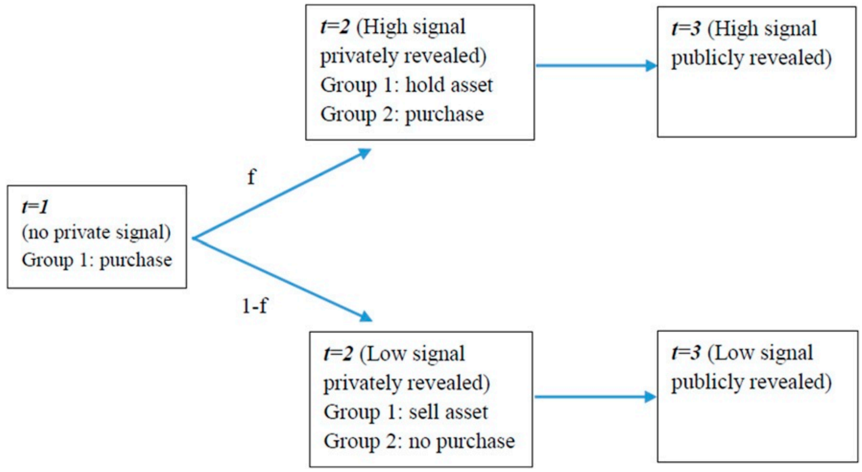

Fig. 1. Timeline of signals and trades by group.

The Equilibrium. Denote the total number of sophisticated traders in group i as x_i, where $i = 1$, 2, and 3, respectively, for groups 1, 2, and 3. For each member in group i, the trader's demand in each period is either 0 or 1. As all members of group i have the same demand, the total demand of group i will then be either 0 or x_i in each period. Denote the demand by group i in period 1 for the asset, as $x_{i,1}$; and denote the demand by group i in period t ($t = 2$, 3), given a private signal s (where $s = h$ or l), as $x^s_{i,t}$. Denote the price in period 1 as p_1; and denote the price in period t ($t = 2$, 3) given a signal s as p^s_t. The equilibria are divided into two types: In one type sophisticated traders make positive profits; in the other type of equilibrium, all sophisticated traders make zero profits and some traders are in group 3 (they buy no assets).

Let us begin with the positive profit equilibrium, with a small number of sophisticated traders, N, so that sophisticated investors make positive profits.

The method of proof will be to specify five (linear) equilibrium conditions with five endogenous variables. Those variables will be respectively the number in group 1 (x_1) who buy the asset in period 1; the number in group 2 (x_2) who buy the asset in period 2 after observing a high signal; and the price of the asset in period 1 (p_1), the price of the asset in period 2 $\left(p^h_2\right)$ given a high signal (h), and the price of the asset in period 2 $\left(p^l_2\right)$ given a low signal (l).

Corresponding to these five endogenous variables, there are five equilibrium conditions to the model. Those equations represent the following:

1) The demand for the asset must be equal to the supply in period 1.
2) In period 2, the demand for the asset must be equal to the supply when a high signal (h) has been privately observed.
3) In period 2, the demand for the asset must be equal to the supply when a low signal (l) has been privately observed.
4) As an arbitrage condition, the profits for those in group 1 must equal the profits for those in group 2.
5) Those who choose group 1 and those who choose group 2 add up to the population of sophisticated investors, N, as long as there is a positive profit to being in group 2.

This leads us to *Proposition 1*, which characterizes the equilibrium when the number of sophisticated investors N is not large.

Small N case. First, we introduce the competition between sophisticates in group 1 and sophisticates in group 2.

Proposition 1. *If* $N < b(1-f)(h+l)$*, then*

1) *The number of sophisticates by group is*

$$x_1 = fN, x_2 = (1-f)N; \quad \textbf{[3]}$$

2) *and the price in period* 1 *will be*

$$p_1 = \hat{\theta} + f\frac{N}{b}; \quad \textbf{[4]}$$

prices in period 2 *will be*

$$p^h_2 = \hat{\theta} + \frac{N}{b};\ p^l_2 = \hat{\theta}. \quad \textbf{[5]}$$

Proof. We will generate the five equations with the five unknowns (while specifying the boundary conditions under which these equations hold).

1) Demand equals supply in period 1. From this condition we find

$$p_1 = \hat{\theta} + \frac{x_1}{b}. \quad \textbf{[6]}$$

Eq. **6** follows from

$$D^u + x_{1,1} + x_{2,1} = 1. \quad \textbf{[7]}$$

The demand for assets by group 1 in period 1, $x_{1,1}$, will be x_1 (since group 1 by definition is buying in period 1). Similarly, the demand for assets by group 2 in period 1, $x_{2,1}$, will be 0 (in this case, since group 2 by definition is not purchasing assets in this period). The market-clearing condition for demand to equal supply (including demand by unsophisticated traders, Eq. **1**) will be

$$1 - b\left(p_1 - \hat{\theta}\right) + x_1 + 0 = 1, \quad \textbf{[8]}$$

whence [**6**] follows.

Eq. **6** makes intuitive sense: If there are x_1 group-1 buyers in period 1, the price will be bid up above fundamentals by the amount necessary for unsophisticated traders to sell them the x_1 units they demand.

2) Demand equals supply in period 2 with a high signal h if

$$p^h_2 = \frac{x_1 + x_2}{b} + \hat{\theta}, \quad \textbf{[9]}$$

and also if the following boundary condition holds: $p^h_2 \leq (1+h)$.

Eq. **9** follows from

$$D^u + x_{1,2}^h + x_{2,2}^h = 1, \quad \textbf{[10]}$$

given the boundary condition. For $p_2^h \le (1+h)$, $x_{1,2}^h = x_1$, and $x_{2,2}^h = x_2$. By assumption, members of groups 1 and 2 will demand the asset in period 2 if the price in that period is less than or equal to the price in period 3, which will be $(1+h)$ in the event of a positive signal h.

Eq. **9** makes intuitive sense: If there are x_1 group-1 buyers in period 2 and x_2 group-2 buyers in period 2, the price will be bid up above fundamentals by the amount necessary for unsophisticated traders to sell them the $x_1 + x_2$ units they demand. In [**9**], the sophisticated traders have bid up the price by $\frac{1}{b}$ times their demand. Their demand is the sum of the number of group 1 and group 2 traders; all of them will be wanting to hold the asset in period 2 with a positive signal, in anticipation that the price will rise yet farther (or remain constant) in period 3, when the positive signal h is publicly revealed.

3) Demand equals supply in period 2 with a low signal l if

$$p_2^l = \hat{\theta}, \quad \textbf{[11]}$$

and also if the following boundary condition holds: $p_2^l > (1-l)$.

Eq. **11** follows from

$$D^u + x_{1,2}^l + x_{2,2}^l = 1, \quad \textbf{[12]}$$

given the boundary condition. For $p_2^l > (1-l)$, $x_{1,2}^l = 0$, and $x_{2,2}^l = 0$. Members of groups 1 and 2 will have 0 demand for the asset in period 2 if the price in that period is larger than the price in period 3, which, by assumption, will be $(1-l)$ in the event of a negative signal l.

For a simple reason, the price of the asset in [**11**] returns to the expected fundamental value $\hat{\theta}$: Group 1 sells the holdings they had acquired in period 1. Those purchases had driven the price up; now the sale of those assets will drive the price down by an equal and opposite amount.

4) The arbitrage condition of equal profits for those in group 1 and group 2.

The expected profit for a member in group 1 is

$$\pi_1 = -p_1 + f(1+h) + (1-f)p_2^l, \quad \textbf{[13]}$$

if the following boundary condition holds: $x_1 > 0$.

The expected profit for a member in group 2 is

$$\pi_2 = f\left((1+h) - p_2^h\right), \quad \textbf{[14]}$$

if the following boundary condition holds: $x_2 > 0$.

In equilibrium, conditional on $x_1 > 0$ and $x_2 > 0$, the expected profit for a member in group 1 should be equal to that in group 2. Combining [**6**], [**9**], [**11**], [**13**], [**14**], and $\pi_1 = \pi_2$ leads to

$$-\frac{x_1}{b} - \hat{\theta} + f(1+h) + (1-f)\hat{\theta} = f\left((1+h) - \frac{x_1 + x_2}{b} - \hat{\theta}\right), \quad \textbf{[15]}$$

so that

$$\frac{x_1}{x_1 + x_2} = f. \quad \textbf{[16]}$$

That is, among sophisticated traders, a fraction f will purchase the asset in period 1, while a fraction $(1-f)$ sophisticated traders will buy the asset only in period 2 after observing a high signal.

At this point we can see why group 1 members have decided to buy in the first period through Eq. **16**. The intuition is straightforward. Relative to group 2, group 1 incurs both an expected loss of $\frac{(1-f)x_1}{b}$ due to selling on occurrence of a low signal in period 2 and an expected gain of $\frac{fx_2}{b}$ from buying in period 1 (rather than in period 2) on occurrence of a high signal in period 2. In equilibrium, the expected loss should be equal to the expected gain: $\frac{(1-f)x_1}{b} = \frac{fx_2}{b}$, which gives Eq. **16**.

5) As long as there are positive profits for group 2, there will be no members of group 3, since they make no trades and have no profits. In this case with positive profits, those who choose group 1 and those who choose group 2 add up to the population of sophisticated investors, N:

$$x_1 + x_2 = N. \quad \textbf{[17]}$$

We can verify that the five variables x_1, x_2, p_1, p_2^h, p_2^l as specified in *Proposition 1* meet the five conditions above: [**6**], [**9**], [**11**], [**16**], and [**17**], along with five associated boundary conditions accompanying [**9**], [**11**], [**16**], and [**17**]: $p_2^h \le (1+h)$, $p_2^l > (1-l)$, $x_1 > 0$, $x_2 > 0$, and $\pi_2 > 0$.

Therefore, an equilibrium as specified in *Proposition 1* exists.

It remains to prove uniqueness. Within the boundary conditions specified, the values of x_1, x_2, p_1, p_2^h, and p_2^l must be unique, since Eqs. **6**, **9**, **11**, **16**, and **17** are a system of linear equations with full rank.

It is also easy to verify (as shown in *SI Appendix*, section A) that no equilibrium could exist in which any of the five boundary conditions is violated. Therefore the values for x_1, x_2, p_1, p_2^h, and p_2^l are not only in equilibrium; that equilibrium is also unique. Q.E.D.

Discussion

The following two corollaries follow easily from *Proposition 1*:

Corollary 1. Size of Deviation of Period 1 Price, p_1, from Fundamentals. *With $N < b(1-f)(h+l)$, the first period deviation of price from fundamentals is increasing in f.*

Proof. The proof follows from calculations of $\left(p_1 - \hat{\theta}\right) = fN$ from [**4**].

Remark. In our model, the initial price deviation from fundamentals increases with the probability of the good state, as frontrunners will be more likely to keep the asset (gain from frontrunning) and less likely to sell the asset (loss from frontrunning).

Corollary 2. Trade Volume. *Trade volume in period 1, x_1, with $N < b(1-f)(h+l)$, is increasing in f.*

Proof. The proof follows from calculations of $x_1 = fN$ from [**3**].

Remark. In our model, as f increases, a larger percentage of sophisticates will purchase earlier to outrun other sophisticates, causing the trade volume in period 1 to increase.

The model may be applied to currency or bond markets, in which a central bank is leaning against the wind to stabilize the market. In these cases, the unsophisticated traders may then represent the central bank; sophisticated traders might include hedge funds or proprietary trading desks at banks. The central bank purchases currency or bonds when prices decline and sells when prices increase, with the goal of stabilizing the market (51–56). The hedge funds seek to profit by trading on private information, and they can make profits because the central bank is willing to lose money to reduce market volatility. Meanwhile, hedge funds face their own limits on arbitrage and may not drive

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prices to their expected value.§§ A tightening of these limits, such as by imposing leverage constraints, could be loosely interpreted in our model as reducing the number of sophisticates (N). A reduced N delays the incorporation of new information into prices. But it also limits the losses to the central bank, by reducing 1) the value loss from trading with hedge funds (for example, $\left((1+h)-p_2^h\right)$ at period 2 for the good signal) and 2) the potential extra transaction costs due to frontrunning.¶¶

Our model also suggests a possible asymmetric response to positive and negative shocks to fundamentals. Such asymmetries may be important in currency markets with strong upward pressure and also in government bond markets with interest rates close to the zero lower bound.##

An empirical puzzle in asset markets is the joint occurrence of high price and high volume (37). Some models have been proposed to explain the puzzle, including, for example, the convenience yield theory (37) and the difference-of-opinion models with short-sale constraints (18). Our model provides an additional explanation: frontrunning arbitrage among sophisticates. In our model, higher f (and consequently improved fundamentals) can also generate the joint occurrence of high price and high volume.

Our model thus cautions on using trading volume as an indicator of market sentiment (relatively optimistic or pessimistic). For example, Baker and Wurgler (59) note that with short-sale constraints, trade volume can be used to proxy the optimism of irrational investors. Our model suggests that an increased volume may also result from better fundamentals (in terms of higher f).

This leads us next to *Proposition 2*, when $N > b\,(1-f)(h+l)$. In this case, the number of sophisticates is sufficiently large that p_2^h is driven to its upper boundary of $(1+h)$. This means that prices will be bid so high that profits to both groups 1 and 2 will both be 0. Nonetheless, we will still see—through the arbitrage condition **16**—that x_1, the number of sophisticated purchasers in period 1, will be positive. At the same time, there will be some sophisticates who will make investments in neither period 1 nor period 2: The number of investors in group 3, x_3, will be positive.

Large N case.

Proposition 2. *When* $N > b\,(1-f)(h+l)$,

1) *the number of sophisticates by group will be*

$$x_1 = b\,(h+l)f\,(1-f),\; x_2 = \frac{(1-f)x_1}{f},\; x_3 = N - x_1 - x_2. \quad \textbf{[18]}$$

2) *And the price in period 1 will be*

$$p_1 = \hat{\theta} + f\left((1+h) - \hat{\theta}\right); \quad \textbf{[19]}$$

and prices in period 2 will be

$$p_2^h = (1+h);\; p_2^l = \hat{\theta}. \quad \textbf{[20]}$$

Proof. See *SI Appendix*, section B.

Remark. According to Eq. **19**, the price deviation in period 1, $f\left((1+h)-\hat{\theta}\right)$, depends on two components. The first component, $(1+h)-\hat{\theta}$, is the gap between the upper-state value at period 3, $(1+h)$, and the expected fundamental value of the asset perceived at period 1 ($\hat{\theta}$). This is the maximum profit a sophisticate can make with the private information under the short-sale constraint. The second component, f, intuitively measuring the asymmetry in the distribution of price changes, captures the probability that a sophisticate will be able to gain from the private information. In this case with large N, these two components effectively determine the level of frontrunning and price deviation at period 1.

Remark. *Proposition 1* describes the equilibrium with small N $(N < b\,(1-f)(h+l))$ and positive profits. *Proposition 2* describes the equilibrium with large N $(N > b\,(1-f)(h+l))$ and zero profits. This is of course natural: The large value of N has driven the profits of the sophisticates to zero.

This is also a good place to remark on our assertion that frontrunning increases prices above fundamentals. This is unambiguously true in period 1, in which $p_1 = \hat{\theta} + \frac{x_1}{b}$. In period 2, both p_2^h and p_2^l will be unchanged if frontrunning is not permitted (i.e., if x_1 is restricted to be 0). In period 3, the price will be equal to the revealed fundamentals. In this sense, frontrunning, period by period, either increases price relative to fundamentals or makes no difference.

Transaction Costs

This section introduces into the core model transaction costs incurred by sophisticated traders in the form of a cost c per share traded (paid by sophisticated buyers and sellers). In the absence of transaction costs, there will be no additional losses due to frontrunning. But with these transaction costs, there will be such losses. This section therefore extends our basic model to include c.

It is easy to understand why there will be no additional losses even to unsophisticated traders from frontrunning if c is 0: 1) In the absence of transaction costs, our model is zero sum: That implies that the losses to unsophisticated traders are exactly equal to the gains to sophisticates. In the absence of transaction costs, with large N, sophisticates make zero profits. Therefore, unsophisticated traders make zero losses. 2) The case of small N is a bit trickier. There are positive profits to sophisticates and therefore losses to unsophisticated traders. But the arbitrage condition ensures that there is no change in the total profits because of the frontrunning (as can be checked, by calculating π_1 and π_2 with and without frontrunning, the total expected profits to sophisticated traders will be $fN\left((1+h)-\frac{N}{b}-\hat{\theta}\right)$).

However, in the presence of transaction costs, frontrunning does bring losses to unsophisticated traders. Accordingly, this section will analyze our model extended to include positive transaction costs.

We first describe (in *Proposition 3*) the equilibrium of the extended model with the presence of a transaction cost c, which we will assume for convenience to be always paid by sophisticated traders. We then characterize (*Proposition 4*) the losses to unsophisticated traders from frontrunning. As before, but with slight modification, sophisticated traders can be partitioned into three groups according to their trading strategies.

§§The limits can be related to the fragile capital structure of hedge funds, their costs of intermediation, and regulations regarding leverage (57, 58).

¶¶As shown later in *Proposition 4* with transaction costs, the extra frontrunning-related loss would be $2c\,(1-f)x_1$, which declines with N. Note also that the model studies only the monetary gains and losses to traders. If one takes into account the utility functions of traders, then the welfare interpretations may vary with assumptions regarding the specific utility functions. For example, if a central bank stabilizes the exchange rate, then there may be welfare gains from this behavior if the central bank's intervention somehow benefits the economy in an unmodeled manner.

##Empirical work has documented some facts consistent with such asymmetries. For example, with currency interventions for 33 countries from 1995 to 2011, Fratzscher et al. (55) have shown that 69% of sales of domestic currency by the central bank have been preceded by such sales the day before; in contrast, only 47% of purchases of domestic currency have been preceded by such purchases the day before. A standard explanation for this pattern is that the central bank is more inclined to depress the currency value to boost exports. Our model suggests the possibility of an alternative explanation: If the upper state of the domestic economy is more likely to occur (i.e., a higher f), then the frontrunning purchase of domestic currency by hedge funds will be more likely, which will then cause the central bank to sell the domestic currency more frequently.

The space of c and N is divided into three parts. There are values of c relative to N, which are sufficiently large that it does not pay to make the two transactions involved in buying in period 1 and selling in period 2. That is, $x_1 = 0$, so there is no frontrunning. But for c sufficiently small relative to N, there will be frontrunning. *Proposition 3* characterizes the equilibria for c sufficiently small and also the area when frontrunning does and does not occur. Furthermore, the space of c and N for which there is frontrunning will be divided into two areas: For lower values of N, there will be positive profits to sophisticates; for higher values of N, there will be zero profits.

Proposition 3. *There exists a unique equilibrium in the asset market*:

1) Case 1: frontrunning with profits (small c $\left(c < \min\left[\frac{f\left((1+h)-\hat{\theta}\right)}{(2-f)}, \frac{fN}{2b(1-f)}\right]\right)$ and small N $\left(N < b\left((1+h)-\hat{\theta}-c\right)\right)$).

The number of sophisticates by group, and the prices, will respectively be

$$x_1 = fN - 2b\,(1-f)c;\ x_2 = (1-f)N + 2b\,(1-f)c;\ \ x_3 = 0;$$

and

$$p_1 = \hat{\theta} + \frac{x_1}{b};\ p_2^h = \hat{\theta} + \frac{x_1 + x_2}{b};\ p_2^l = \hat{\theta}.$$

2) Case 2: frontrunning without profits (small c $\left(c < \min\left[\frac{f\left((1+h)-\hat{\theta}\right)}{(2-f)}, \frac{fN}{2b(1-f)}\right]\right)$ and large N $\left(N > b\left((1+h)-\hat{\theta}-c\right)\right)$).

The number of sophisticates by group, and the prices, will respectively be

$$x_1 = b\left(f\left((1+h)-\hat{\theta}\right) - (2-f)c\right);$$
$$x_2 = (1-f)b\left(\left((1+h)-\hat{\theta}\right) + c\right);$$
$$x_3 = N - x_1 - x_2;$$

and

$$p_1 = \hat{\theta} + \frac{x_1}{b};\ p_2^h = \hat{\theta} + \frac{x_1 + x_2}{b};\ p_2^l = \hat{\theta}.$$

3) Case 3: no frontrunning (large c $\left(c > \min\left[\frac{f\left((1+h)-\hat{\theta}\right)}{(2-f)}, \frac{fN}{2b(1-f)}\right]\right)$).

The number of sophisticates in group 1 will be

$$x_1 = 0.$$

Fig. 2 pictures the division of the three cases according to the values of the parameters, N and c.

Proof. See *SI Appendix*, section C. The basic logic follows the proofs of *Propositions 1* and *2*. The major modifications occur with the respecification of the arbitrage conditions between group 1 and group 2 and between group 2 and group 3 to include the transaction costs c, as well as the respective boundary conditions.

Remark. The results of *Proposition 3* are intuitive. It does not pay to do the two transactions involved in frontrunning if the transaction cost c is sufficiently large. (This occurs if $c > \min\left[\frac{f\left((1+h)-\hat{\theta}\right)}{(2-f)}, \frac{fN}{2b(1-f)}\right]$). But for a smaller cost c, it will pay to make those two transactions. Again, as in *Propositions 1* and *2*, with large N, there will be zero profits; with small N, there will be positive profits to sophisticates.

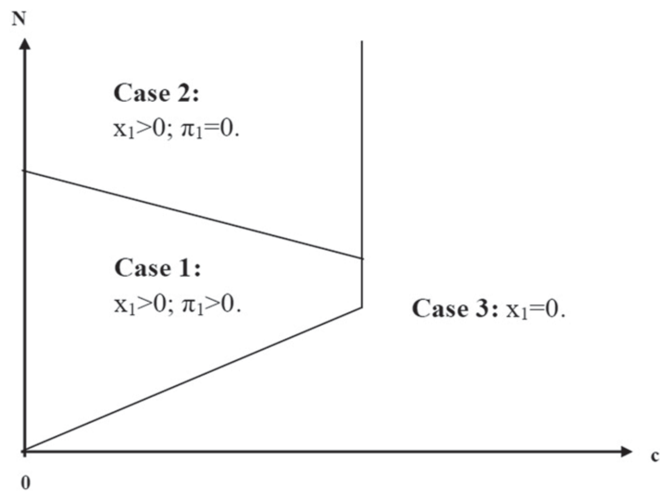

Fig. 2. Partition of equilibrium types by c and N.

Our model suggests that a Tobin tax can have a large effect on frontrunning. The tax affects the trade-off between buying earlier versus buying later, as early purchasers may incur transaction costs twice—both when they purchase the asset in period 1 and when they sell it in period 2 after observation of a low signal. As shown in *Proposition 3*, if $c > \min\left[\frac{f\left((1+h)-\hat{\theta}\right)}{(2-f)}, \frac{fN}{2b(1-f)}\right]$, then there will no frontrunning and hence no price deviation in period 1 (i.e., $x_1 = 0$, and $p_1 = \hat{\theta}$). This occurs even though sophisticates may still purchase in period 2 after privately observing a high signal ($x_2 = \min\left[\max\left[\frac{(1+h)-\hat{\theta}-c}{b}, 0\right], N\right]$).

We now come to the major point of this section, which is to evaluate the losses from frontrunning.

Proposition 4. *The loss of unsophisticated traders due to frontrunning will be*

$$Loss_{frontrunning} = 2c\,(1-f)x_1. \qquad \textbf{[21]}$$

Proof. The proof, which is in *SI Appendix*, section D, follows from calculating the losses to the unsophisticated traders according to the values of x_1, x_2 and p_1, p_2^h, p_2^l as specified in *Proposition 3*. (It is assumed that the value of holding the asset in period 3 both to sophisticates and to unsophisticated traders will be the respective fundamental value, dependent on whether the signal is h or l. There will be no trade in this period if there is a transactions cost, as traders would receive less than the fundamental value of the asset.)

The result in **[21]** is intuitive. This intuition, however, varies according to cases 1, 2, and 3 in *Proposition 3*. In case 2, sophisticates make no profits: Since this is an equilibrium of a zero-sum economy, the loss to unsophisticated traders will be the expected increase in transaction costs due to frontrunning. Since the frontrunning increases the expected trade volume by $2\,(1-f)x_1$, those expected losses to unsophisticated traders due to frontrunning will increase by $2c\,(1-f)x_1$.

Case 1, with profits, is a bit more complex. Note, however, that in the equilibrium of case 1, in the event of a positive signal, p_2^h rises to $\frac{N}{b}$ above fundamentals, $(\hat{\theta} + \frac{N}{b})$. This price will be exactly unchanged in the absence of frontrunning, since again in period 2, there will be N sophisticated holders of the asset. The arbitrage condition also causes group 1 and group

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2 to earn the same expected profits. Since frontrunning leaves group 2's expected profits untouched, group 1's expected profits will also be the same. The only change in the expected loss to unsophisticated traders must then come one for one from the added transaction costs due to the frontrunning by the group 1 traders. Those increased transaction costs will be the product of the frontrunning-induced trade volume in periods 1 and 2 and the transaction cost c: $2c(1-f)x_1$.

Case 3 is the easiest. There is no frontrunning ($x_1 = 0$). There is no additional loss due to frontrunning.

Remark. Note that all of the added costs of the frontrunning in our model, both in the case of large N (and competitive markets) and in the case of small N (and profits to sophisticated traders), are totally borne by the unsophisticated traders. In all cases, the profits to the sophisticates are unchanged. (This exact result of course depends on our specific designation of the model.)

Remark. We have so far presented a barebones model. In more complicated models, one could relax the constraint to allow some short sales or to allow some inference of signals by unsophisticated traders. To the extent that the existence of unsophisticated traders allows a sophisticate to make positive profits from frontrunning, our mechanism will still carry through. In *SI Appendix*, section E, we show that if the second-period price does not fall all of the way to its fundamental value (i.e., $1-l$) on receipt of a private negative signal—for example, due to limited capacity of short sellers or imperfect inference of unsophisticated traders, then with low transaction costs there will still be equilibrium frontrunning of the signal. The arbitrage condition will be modified, as well as the equilibrium number of frontrunners, to reflect the gains and losses to frontrunners. Nonetheless, our main results on gains and losses will remain. That is because the arbitrage condition equalizes the returns of the frontrunners and those who wait for the signal. To the extent that the total information advantage of sophisticated traders is fixed relative to that of unsophisticated traders, then the additional cost of frontrunning will still be borne wholly by the unsophisticated traders, with no gain to the sophisticates.

Sequential Signals

In the previous models, there is a single signal about the fundamental value of the asset. This section considers the possibility of sequential signals. It will show that frontrunning enhances the initial price relative to fundamentals: The deviation from fundamentals (in the initial period) will be a damped sum of the frontrunning effects from the individual sequential signals. Relative to our baseline model, these additive properties are further reason why frontrunning amplifies trade volume and also the loss to unsophisticated traders.

In our modified model, we assume that there are two sequential signals and five periods: $t = 1, 2, 3, 4, 5$. There will be two independent ingredients of the asset's value. One of these, η_a, is fully revealed in period 3; the other, η_b, is fully revealed in period 5. The fundamental value of the asset in period 5 is then

$$\theta_5 = 1 + \eta_a + \eta_b.$$

η_a has the following two-point distribution: With probability f_a of being h_a, and probability $(1-f_a)$ of being $-l_a$. η_a is privately observable in period 2; it will be publicly announced in period 3. η_b has the following two-point distribution: With probability f_b of being h_b, and probability $(1-f_b)$ of being $-l_b$. η_b is privately observable in period 4; it will be publicly announced in period 5. Sophisticates with access to η_a are type A and sophisticates with access to η_b are type B.

The signature result of our previous model has been that the transaction costs resulting from the frontrunning were absorbed totally by the unsophisticated traders with no gains to sophisticates, because the profits of the sophisticates, who are the nonpassive actors in the respective markets, are anchored. This result will continue to hold in our extended isomorphic model with sequential signals. This proposition follows from backward induction. Going back from period 5 to period 3, that same proposition holds exactly. But then going back inductively from period 3 to period 1, it will hold yet again for the exact same reason. A detailed version of the proposition is given in *SI Appendix*, section F.

Moreover, the sequential arrival of signals aggravates the loss from frontrunning. In a benchmark case of two identical sequential signals, the total loss from frontrunning the two signals more than doubles the loss from frontrunning a single signal. For instance, with $f_a = f_b = \frac{1}{2}$, $h_a = l_a = h_b = l_b = z$, and large N_a and N_b, then the loss from frontrunning the two signals will be $\frac{5}{2}$ of the loss for a single signal (*SI Appendix*, section F).[The result is a little subtle. The frontrunning-related loss of unsophisticated traders is $\frac{5b(z-3c)c}{4}$, as the sum of the loss due to the frontrunning by type A sophisticates (i.e., $\frac{3b(z-3c)c}{4}$) and the loss due to the frontrunning by type B sophisticates (i.e., $\frac{b(z-3c)c}{2}$). The loss due to type A sophisticates is $\frac{3}{2}$ of that due to type B sophisticates because type A sophisticates not only frontrun on η_a but also half of the time frontrun on type B's frontrunning of η_b (type A sophisticates must consider the probability $\frac{1}{2}$ that by period 3 they will have already sold the asset due to a low value of η_a revealed to them at period 2). For comparison, the frontrunning-related loss in the case of a single signal is $\frac{b(z-3c)c}{2}$.]

Conclusion

This paper has added another dimension to the analysis of equilibria in markets with unsophisticated traders. It explains why sophisticated investors might purchase an asset before, rather than after, the receipt of information.

Indeed, the heart of our model is an arbitrage equation whereby sophisticated investors purchase assets in advance of receiving information to avoid the loss due to waiting too long if the information turns out positive; as they also take advantage of the opportunity to dump if the information turns out negative. This equilibrium does not just involve the interaction between unsophisticated traders and sophisticates. Instead, in the spirit of Shleifer and Vishny (8), its example of frontrunning gives further demonstration of critical interactions between different groups of sophisticates themselves in unsophisticated-trader models.

Our model generates a special result: All of the transaction costs of the extra frontrunning are borne by the unsophisticated traders with no gain to the sophisticates. This paper hence provides a specific instance of inefficient financial transactions and excessive rent seeking with gains to no one. Nor does the frontrunning produce any information discovery.

Data Availability. There are no data underlying this work.

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